

01 Natural numbers

BAR CODE

Every item we find in a supermarket has a number to identify it quickly. This is the bar code.

The bar code of everything has usually 13 digits. The bars are a way to introduce these numbers through a laser reader.



THE MEANING OF A BAR CODE

We are going to do this activity with the above example. Fill the gaps with the bar code.

[illegible]

WHAT IS THE INFORMATION OF A BAR CODE?

The first two, sometimes the first three, figures are the country code.

The following five digits are the information about the company.

The following five digits are about the product and, finally, the last digit is the control digit which is the interesting subject for us in this chapter.

WHAT IS THE PURPOSE OF THE CONTROL DIGIT?

The purpose of the control digit is to find possible reading or writing mistakes by a machine.

The control code has a formula that links its value with the other numbers.

FORMULA

To get the control digit you have to do the following operations:

Add the digits which are in odd position from left to right not including the last digit because it is the control digit.

Add up the even digits from left to right and after that multiply the result by 3.

After that we add the two results.

Finally, the control digit is the number we have to add in order to get a multiple of ten.

In this case it is 7.

[illegible]

There are other numbers to identify something or someone. For instance, the identity card, the bank account number, the international standard book number (ISBN)...

1. NUMBERS

A number is the symbolic expression of a quantity.

For example, 5 books, 1.75 m; XVI century.

A digit is each symbol we use to make numbers.

Our number system is named decimal because it has 10 digits: 0...9

Our number system is positional because the value of a digit depends on the position that it has.

For example, $543 = 5 \cdot 100 + 4 \cdot 10 + 3$

We represent the numbers in a line with two values: the zero and the unit. This is the number line.

Numbers are used to describe quantities but order too.

The symbols we use to express order are '<' and '>'.

For example, $2 < 5$ (two is less than five); $7 > 3$ (seven is greater than three).

2. NATURAL NUMBERS

Natural numbers are the easiest numbers.

They are used to count.

We represent these by the capital letter N: 1, 2, 3...

They are infinite because they never end.

3. ADDITION AND MULTIPLICATION

Addition

Addition is the operation that gives the value of the union of two quantities.

For example, 7 euros + 5 euros = 12 euros.

Addition has two properties: commutative and associative.

Commutative means that the order of the quantities doesn't affect the result.

That is, $7 + 5 = 5 + 7$

Associative means that if we have to do two additions we can associate numbers in every way.

For example, $(3 + 4) + 8 = 3 + (4 + 8)$

Subtraction is the inverse operation.

It is the result of taking away some quantity from another.

For example, 7 euros – 5 euros = 2 euros.

Multiplication

Multiplication is the way to represent a multiple addition.

For example, $5 \times 3 = 3 + 3 + 3 + 3 + 3$.

This operation is commutative and associative too.

$7 \times 5 = 5 \times 7$

$(3 \times 4) \times 8 = 3 \times (4 \times 8)$

The inverse operation of multiplication is division. Therefore, it is the result of a multiple subtraction.

Distributive property

And ~~finally~~finally, we have a property that links addition and multiplication. It is the distributive property.

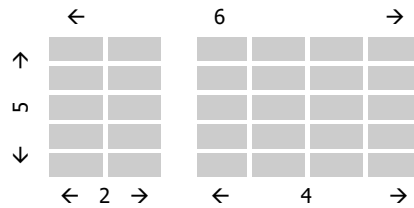
Con formato: Español (España)

The meaning of this property is the following:

To multiply a number by a sum or subtraction we have to multiply the number by each member.

$$5 \cdot (2 + 4) = 5 \cdot 2 + 5 \cdot 4$$

You have two ways to calculate the number of pupils in a class: multiplying the number of rows by the total of columns or multiplying each part of the class separated by the aisle.



The formula is the following: that is, for whatever numbers:

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

4. ORDER OF OPERATIONS

To do combined operations we have to follow the next order:

First. We do brackets.

After. Multiplication and division.

After. Addition and subtraction.

Operations with same hierarchy we do from left to right.

Example

$$5 \cdot (7 - 2) + 2 \cdot (2 + 4 \cdot 3) - 5 =$$

5. SALES PROBLEMS

It is important to understand these three concepts to solve problems:

Expenses: The money I use to buy products to sell.

Profit: The money I get as a result of the sale.

Sale price: The money that the customer pays in the shop.

The relationship among them is the following:

$$\text{SALES PRICE} = \text{EXPENSES} + \text{PROFIT}$$

Example

A shopkeeper bought 850 pairs of trousers for €9 each. He sells all of them getting €12750 for the sale. What is the profit per pair of trousers?

EXPENSES –total-:

PROFIT –total-:

SALES PRICE – total-:

THEORY SUMMARY

Number

A number is the symbolic expression of a quantity.

Digit

A digit is each symbol we use to make numbers.

Number system properties

Our number system is named decimal because it has 10 digits: 0...9

Our number system is positional because the value of a digit depends on the position that it has.

Order

The symbols we use to express order are: '<' -less than- and '>' -greater than-

Natural numbers

The numbers we use to count. We represent these by the capital letter N: 1, 2, 3...

Commutative

Commutative means that the order of the quantities in an addition or multiplication doesn't affect the result.

Associative

Associative means that if we have to do two additions or multiplications we can associate numbers in every way.

Distributive

To multiply a number by a sum or subtraction we have to multiply the number by each member.

Sales formula

The relationship among the expenses, the profit and the sale price is the following:

$$\text{SALE PRICE} = \text{EXPENSES} + \text{PROFIT}$$

EXERCISES AND PROBLEMS

1. NUMBERS

2. NATURAL NUMBERS

3. ADDITION AND MULTIPLICATION

- 1.** Put the result of these operations in each square:

[illegible]

- 2.** Write the answer to the question in each cell:

[illegible]

- 3.** Do the same as above:

[illegible]

- 4.** Obtain the result of each step for the next chain.

[illegible]

- 5.** Calculate mentally, do not write anything. You have to spend less than one minute on getting the final result.

Twelve	subtract eight	multiplied by six	plus one	divide by five	plus two	multiplied by four	plus two	half of this	minus six
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- 6.** Work out in your mind following the operations in each step. You have to get the final result in less than one minute.

Seventeen	take away five	divided by four	eight times this	double it	add one	divided by seven	times eight	plus four	half of it
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7. Do the same as the previous ones:

Thirty nine	plus nine	divide by eight	times itself	minus one	divide into five	multiplied by four	remove one	share into three	multiplied by seven

8. Calculate mentally. This is without using the pen. After that, write the final result.

33	add fifteen	$\frac{1}{4}$ of this	one sixth of this	multiplied by 9	times 2	6 more	one seventh of this	times nine	minus 5

9. Do the following multiplication: 37569×478
10. Do the following multiplication: 402987×673
11. Calculate the quotient and the remainder for the division (N is your list number):
a) $N43596:9$; b) $N375678:9$; c) $N717372:9$
12. Calculate the quotient and the remainder for the division (N is your list number):
a) $N54391:72$; b) $N107964:54$; c) $N436932:63$; d) $N5764770:54$
13. Calculate the quotient and the remainder for the division (N is your list number):
a) $N576477:954$; b) $N702594:756$; c) $N355986:837$
14. Find the quotient and the remainder of this division (N is your list number):
 $N730485:864$
15. Do the following division, that is, calculate its quotient and its remainder (N is your list number): $N246384:729$

4. ORDER OF OPERATIONS

16. Do the following combined operation: $23 + 2 \cdot (18 - 6 : 3) - 5 \cdot 4$
17. Do step by step: $13 \cdot 2 \cdot 5 - 4 \cdot (8 - 6 : 2) - 3 \cdot 5$
18. Work out the following operation: $3 + 2 \cdot (15 - 5 \cdot 2) - 2 \cdot 3$
19. Calculate doing the parenthesis first:
a) $7 \cdot (3 + 9) =$; b) $12 \cdot (5 + 8) =$; c) $12 \cdot (10 + 8) =$
20. Calculate applying the distributive property:
a) $7 \cdot (3 + 9) =$; b) $12 \cdot (5 + 8) =$; c) $12 \cdot (10 + 8) =$
21. Look at this way to calculate using the distributive property:
 $15 \cdot 18 = 15 \cdot (10 + 8) = 150 + 120 = 270$.
Do the same to calculate:
a) $12 \cdot 15 =$; b) $17 \cdot 24 =$; c) $25 \cdot 32 =$

5. SALES PROBLEMS

22. A shopkeeper bought 1200 pairs of trousers for € 35 each. He sells all of them getting € 44400 profit. What is the gain per pair of trousers? What is the sales price per unit?
23. A woman carries 15 vases to sell them in a market for € 12 each. On the way she has an accident and 3 vases break. How much must the price be to get the same money?

- 24.** A dealer carries 12 vases to sell them in a market for € 30 each one. On the way he has an accident and 2 vases break. How much must be the price to get the same money?
- 25.** A carrier has 20 mirrors that he plans to sell at € 24 each. 4 mirrors are broken in an accident. At what price does he have to sell the remaining mirrors to obtain the same amount of money?
- 26.** A farmer buys a cow for € 1100. The cow consumes a sack of feed per day which costs € 2. Every day the farmer gets 40 litres of milk, which sells for € 0.5 per litre. He sells it for € 900 after 180 days. What benefit has he obtained?
- 27.** A trader bought 500 boxes of tomatoes 10 kg each for 4500 euros in total. The transport cost 600 euros. On the way he lost 500 kg of tomatoes. What will the price be per kilo to get 3900 euros as profit?
- 28.** Place among the numbers the signs +, -, x and parenthesis as you want in order to fit each equality. For example, $4 \times (3 + 2 - 1) = 16$

4 3 2 1 = 1	4 3 2 1 = 6	4 3 2 1 = 11
4 3 2 1 = 2	4 3 2 1 = 7	4 3 2 1 = 12
4 3 2 1 = 3	4 3 2 1 = 8	4 3 2 1 = 13
4 3 2 1 = 4	4 3 2 1 = 9	4 3 2 1 = 14
4 3 2 1 = 5	4 3 2 1 = 10	4 3 2 1 = 15

- 29.** What is the control digit for the following bar code: 8 4 1 2 3 7 6 2 8 7 9 5 __
- 30.** Calculate the control code for the following bar code: 5 9 2 3 6 1 4 6 0 6 0 3 __